Automatic wave-equation migration velocity analysis by focusing subsurface virtual sources

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ABSTRACT

Macro-velocity model building is important for subsequent prestack depth migration and full-waveform inversion. Wave-equation migration velocity analysis uses the band-limited wavefield to invert for velocity. Normally, inversion would be implemented by focusing the subsurface offset common-image gathers. We reexamine this concept with a different perspective: In the subsurface offset domain, using extended Born modeling, the recorded data can be considered as invariant with respect to the perturbation of the position of the virtual sources and velocity at the same time. A linear system connecting the perturbation of the position of those virtual sources and velocity is derived and solved subsequently by the conjugate gradient method. In theory, the perturbation of the position of the virtual sources is given by the Rytov approximation. Thus, compared with the Born approximation, it relaxes the dependency on amplitude and makes the proposed method more applicable for real data. We determined the effectiveness of the approach by applying the proposed method on isotropic and anisotropic vertical transverse isotropic synthetic data. A real data set example verifies the robustness of the proposed method.

INTRODUCTION

Macro-velocity model building is important in seismic data processing. Prestack depth migration relies on a smooth kinematically accurate velocity model to image the subsurface and full-waveform inversion needs a correct background velocity to avoid cycle skipping and to converge as well (Virieux and Operto, 2009; Alkhalifah, 2016a). Although the seismic migration method has evolved from relying on ray-based methods, such as Kirchhoff migration, to waveform-based methods, such as wave-equation migration and reverse time migration (Etgen et al., 2009), many of the methods for estimating the macrovelocity model in the industry are still based on ray tracing (Jones et al., 2008).

Wave-equation migration velocity analysis (WEMVA) uses the wavefield as a carrier for velocity information. Although a few variations of the WEMVA methods have been proposed over the years, they share the same main features of back propagating the residual images for velocity updating and the difference lies in how to compute such residual images. The residual image can be constructed in the image domain by a penalty function (Shen and Symes, 2008), warping (Perrone et al., 2014), or horizontal concentration (Shen and Symes, 2015). Among these methods, differential semblance optimization (DSO) (Symes, 2008) is the most popular because it relies on expanding the image space and penalizing energy residing in this nonphysical extension. However, the gradient of the DSO operator is severely contaminated by artifacts. Recent developments (Hou and Symes, 2015, 2017; Chauris and Cocher, 2017) demonstrate that an inversion rather than migration for the reflectivity can reduce such artifacts. This demonstrates that the DSO approach is sensitive to the amplitude of the resulting image, requiring “true amplitude” extended imaging.

Anisotropic media assumptions provide more realistic representations of the subsurface in which a complex geologic environment exists. Successfully building the anisotropic model is essential to properly image the subsurface in such media. The vertical transverse isotropic (VTI) (Alkhalifah, 1998) model is representative of the first-order anisotropy present in the subsurface. Several authors have used WEMVA in VTI media to build an anisotropic model based on the DSO method (Li et al., 2014; Weibull and Aarnes, 2014).

In this paper, we reexamine WEMVA method with a different perspective. We specifically represent the energy in the extended image as virtual sources in the subsurface offset domain, and by considering the recorded data to be stationary, we build a linear sys-

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term connecting perturbations in the position of the virtual sources from zero offset to perturbations in the background velocity model. The derived linear system can be efficiently solved using conjugate gradient (CG) methods. The proposed approach does not compute the residual image in the image domain as other WEMVA methods. Instead, it has a demigration step induced by perturbing the subsurface offset virtual sources (SOVSs), and, thus, the residuals are actually in the data space. By proper parameterization of the anisotropic parameters (Alkhalifah, 2016b) combined with an appropriate preconditioning of the inversion process, we also extend the proposed method to update the anisotropic parameters of VTI media.

In the following, we first discuss the theory and then show how to incorporate anisotropic parameters in the inversion. For isotropic and anisotropic migration velocity analysis, we present synthetic and real data examples to verify the proposed method.

THEORY

Isotropic WEMVA

The extended imaging condition (Sava and Fomel, 2003) provides the subsurface offset common-image gathers (SOCIGs) \( I(x, h) \) as follows:

\[
I(x, h) = \int dx d\omega S^*(x - h; x_s, \omega) R(x + h; x_s, \omega),
\]

(1)

where * represents the complex conjugate operation, \( x = (x, y, z) \) is the Cartesian coordinates, \( x_s \) denotes the location of the sources of the seismic experiments, and \( \omega \) is the frequency. The functions \( S \) and \( R \) correspond to the source and receiver wavefields, respectively. The subsurface half-offset is given by \( h \): For two dimensions, it has one component and corresponds to a shift in the \( x \)-direction, \( h = (h_x, 0, 0) \); whereas for three dimensions, it will contain two components with respect to both shifts in the \( x \)- and \( y \)-directions: \( h = (h_x, h_y, 0) \).

Considering the adjoint of equation 1, we obtain the extended Born modeling or demigration (Hou and Symes, 2015, 2017; Chauris and Cocher, 2017) as

\[
d(x_s; x_s, \omega) = \int dx dh G(x - h; x_s, \omega) I(x, h) R(x + h; x_s, \omega),
\]

(2)

where \( G(x_1; x_2, \omega) \) denotes the Green’s function from position \( x_2 \) to \( x_1 \) and \( d(x_1; x_s, \omega) \) denotes the record at receiver position \( x_s \) due to source at position \( x_1 \). Equation 2 can be regarded as a virtual source \( f_l(x, h, \omega) \) at location \( x - h \) with source function

\[
f_l(x, h, \omega) = G(x - h; x_s, \omega) I(x, h),
\]

(3)

where \( l \) is the position for the virtual source \( I(x, h) = x - h \).

The record \( d(x_s; x_s, \omega) \) can be considered invariant with respect to perturbations in the position of the virtual sources and an equivalent perturbation in the velocity model. Considering a perturbation in equation 2 for the velocity \( v(x) \) and virtual source position \( l(x, h) \), we end up with

\[
0 = \delta d(x_s; x_s, \omega) = \delta d_l(x_s; x_s, \omega) + \delta d_v(x_s; x_s, \omega)
\]

\[
= \int dx dh \frac{\partial d_l(x_s; x_s, \omega)}{\partial l(x, h)} \delta l(x, h)
\]

\[
+ \int dx dh \left[ \frac{\partial G(x - h; x_s, \omega)}{\partial v(x)} \delta v(x) I(x, h) G(x; x + h, \omega) \right]
\]

\[
+ G(x - h; x_s, \omega) I(x, h) \frac{\partial G(x; x + h, \omega)}{\partial v(x)} \delta v(x),
\]

(5)

where \( \delta d_l \) and \( \delta d_v \) correspond to the perturbation of the record with respect to virtual source position and the velocity, respectively.

We can rewrite the above equation as

\[
\delta d_v = K_v \delta v = -\delta d_l,
\]

(6)

where \( K_v \) is the sensitivity kernel for perturbation in the velocity. To use the CG method (Shewchuk, 1994) to solve this equation, we apply the adjoint of the operator \( K_v \) and obtain the normal equations for CG

\[
K_v^* K_v \delta v = -K_v^* \delta d_l.
\]

(7)

In Appendix B, we give detailed derivations of the operator \( K_v \) and \( K_v^* \).

For MVA, focusing on the SOCIGs can also be explained as moving the position of the virtual source from \( x - h \) to \( x \); i.e., the perturbation of the source position is the negative value of the subsurface half-offset \( h \):

\[
\delta l(x, h) = -h.
\]

(8)

Here, we assumed that when the velocity is changing toward the correct one, the position of the virtual source shifts toward zero offset, and like MV A, we expect smoothed velocity perturbations. For a more complex medium such as having a small Gaussian lens with a high velocity perturbation, the behavior in the evolution of the virtual source position can be complicated, resulting in the divergence of the approach. According to equation A-14, \( \delta d_l \) can be computed as

\[
\delta d_l = \int dx dh f_l(x, h, \omega) \delta l \cdot \nabla_h G(x_s; x + h, \omega).
\]

(9)

Anisotropic WEMVA

Extending the proposed method to VTI media is straightforward. We need to modify equation 7 to include the anisotropic behavior of waves and the additional anisotropic parameters needed to describe the model:

\[
\delta d_m(x_s; x_s, \omega) = K_m \delta m = -\delta d_l.
\]

(10)
The computation of $\delta d_l$ is the same as for the isotropic case defined in equation 9. In this study, we use the one-way acoustic anisotropic equation (Alkhalifah, 2000) for wavefield and sensitivity kernel computation. The anisotropic parameters used here are the vertical velocity $v_0$, NMO velocity $v_{\text{NMO}}$, and $\eta$ (Alkhalifah, 1998); i.e., $m = (v_0, v_{\text{NMO}}, \eta)$. This parameterization results in reduced tradeoff between different parameters (Alkhalifah, 2016b) in MVA.

For the simultaneous inversion of the anisotropic parameters, because they have different scales, we apply a preconditioning to $\delta m$ in equation 10 as follows:

$$K_m P \delta m = -\delta d_l,$$

where $P$ properly scales the anisotropic parameters to guarantee that all parameters can get updated simultaneously in the inversion process. In synthetic and real data examples, the scaling parameters are chosen as $P_{v_{\text{NMO}}}:P_{\eta} = 2000:1$.

**IMPLEMENTATION OF THE METHOD**

Because the implementations of the isotropic and anisotropic versions of the proposed method are similar, we restrict the discussion to the isotropic WEMVA. Implementation of the proposed method includes inner and outer loop iterations. The outer loop iteration provides the gradient of the velocity perturbation by solving the linear system defined in equation 7. Within every outer loop iteration, inner loop iterations are used in a line search to determine the optimal step length, and we use for that the misfit function defined by the DSO approach; i.e., we seek a step length that minimizes the DSO misfit function. Note that the DSO misfit function is only used for the simultaneous inversion of the anisotropic parameters.
for step length optimization. The gradient computation is based on
the virtual source method not by adjoint analysis of the DSO misfit
function. In the inner loop, and to solve the linear system defined in
equation 7, we need to perform demigration using equation 9 to
obtain the residual data, which is the right-hand side of the linear
system, and the linear system is solved using CG methods. We pro-
cceed with the inner and outer loops until a predefined maximum
iterations number is met or the residual data become relatively small
compared with the initial model.

EXAMPLES

In this section, we show the results of applying the proposed ap-
proach on synthetic isotropic and anisotropic data. These tests are
meant to demonstrate the approach features and limitations. We also
show results from field data to demonstrate robustness.

Tests of isotropic WEMVA

The first example corresponds to a layered model. The size of the
model is 4 km in depth and 6 km laterally. We use Born modeling to
create the data and the maximum offset is 6 km. In Figure 1a, the red
line is the true velocity. The black line corresponds to a highly
smoothed initial velocity. The blue line is the inverted velocity after

Figure 3. WEMVA result of the marine data set: (a) image obtained
using the initial velocity and (b) image obtained using the inverted
velocity.

Figure 4. WEMVA result of the marine data set: (a) ADCIGs using
the initial velocity and (b) ADCIGs using the inverted velocity.

Figure 5. Anisotropic WEMVA result of the layered model: (a) NMO
velocity and (b) $\eta$. 
10 nonlinear iterations. We do not show here that actually two to three iterations would give a relatively good result and flatten the gathers for this synthetic example; however, more iterations improve the inverted model. Note that the inverted velocity has reasonably high resolution and is generally consistent with the true model, which is mainly due to the large number of reflectors in the model in which a Gauss-Newton solution of the linear system would boost the high-frequency component. Figure 1b and 1c shows the SO-CIGs for the initial and the inverted velocity model, respectively. From this result, it is clear that after the WEMVA updating, SO-CIGs becomes well-focused.

The second example is a marine real data set from offshore Australia. The offset range is from 160 to 8200 m. The initial velocity is given in Figure 2a. During the inversion, we low-pass filtered the record data up to 30 Hz and we use a 15 Hz peak frequency Ricker wavelet for the source. The image and the angle-domain common-image gathers (ADCIGs) are shown in Figures 3a and 4a, respectively. We can see that the ADCIGs have residual moveouts (RMOs) indicating the inaccuracy of the initial velocity, and the image is not well-focused as well. Figure 2b shows the inverted velocity after two nonlinear iterations. Figures 3b and 4b show the image and ADCIGs produced with the inverted velocity. Compared with the image produced using initial velocity, the resulting image from our method is far more focused and the ADCIGs are reasonably flat in most regions.

Tests of anisotropic WEMVA

Like the isotropic case, we first test the approach on an anisotropic synthetic layered model. The model size is 7.5 km in depth.
and 21 km laterally. The maximum offset is 10 km. In the inversion, we set $v_0 = v_{NMO}$ and invert for $v_{NMO}$ and $\eta$, simultaneously. Figure 5a and 5b shows the result for the inverted anisotropic parameters after 20 nonlinear iterations. From these results, we see that $v_{NMO}$ is well-resolved and $\eta$ captures the main features in the true model. Figure 6a and 6b shows the SOCIGs for the initial and inverted models, respectively. As expected, the gathers are far more focused to the near subsurface offset after our anisotropic WEMVA.

The second example corresponds to the Volve ocean-bottom cable field data set. The maximum offset is approximately 5 km. During the inversion, we set $v_0 = v_{NMO}$ and invert for $v_{NMO}$ and $\eta$. Figures 7 and 8 show the initial and inverted anisotropic parameters, respectively, using two nonlinear iterations. Figure 9a and 9b shows the image using the initial and inverted models, respectively. It is obvious that the image is better focused using the inverted model, and this, in particular, is clear in the area at approximately 2 km depth. When we look at the ADCIGs, we see better flattening of the image gathers using the inverted model (Figure 10b) compared with the initial model (Figure 10a), especially for the shallow part. For the deeper part, we may see some RMOs, for example, at approximately 3 km. We expect it is due to the relative small offset to depth ratios there.

![Figure 10. Anisotropic WEMVA result of the Volve data: (a) AD-CIGs using the initial model and (b) AD-CIGs using the inverted model.](image)

**CONCLUSION**

We proposed a method for macro-velocity updating by focusing the SOVS. Unlike other WEMVA approaches, the proposed method does not require computing the residual image and instead it uses a demigration step to compute the residual in the data domain. A linear system of equations relating the residual data to velocity perturbation is constructed, and a CG method is used to obtain the solution. The advantage of the proposed method is provided by two factors: The measure of focusing is provided by the virtual source representation rather than penalizing the amplitude, which renders our approach to be extended image amplitude insensitive. In other words, and in theory, the DSO method is based on the Born approximation, whereas the virtual source method is based on the Rytov approximation, which can relax the requirement for true amplitude imaging. The second factor is that we include a demigration step and the residual is measured in the data domain. This would provide more robustness for application on real data. For example, we can apply denoising and demultiples in the image domain prior to demigration to obtain a cleaner residual to extract the gradient. As shown in the real data example, we also extend this method to anisotropic media, specifically, VTI. We use a parameterization that provides a reduced trade-off between the different parameters and uses preconditioning to scale the updates of the anisotropic parameters properly, as they are updated simultaneously. The synthetic and real data examples demonstrate the effectiveness and robustness of the proposed method.
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APPENDIX A

WAVEFIELD PERTURBATION FOR SOURCE PERTURBATION

The acoustic-wave equation in the frequency domain can be expressed as

$$-\frac{\omega^2}{v^2(x)} u(x, \omega) = \nabla^2 u(x, \omega) + f(\omega) \delta(x - x_s),$$  \hspace{1cm} (A-1)

where $\omega$ is the frequency, $f(\omega)$ is the source wavelet, $v(x)$ is the velocity, and $u(x, \omega)$ is the wavefield.

A perturbation $\delta x_s$ in the source position leads to wavefield perturbation $\delta u$. We denote the wavefield computed with source at location $x_s + \delta x_s$ as $u_s(x, \omega)$ and we have

$$u_s(x, \omega) = u(x, \omega) + \delta u(x, \omega).$$ \hspace{1cm} (A-2)

Alkhalifah (2010) points out that the waves generated at the location $x_s + \delta x_s$ with velocity model $v(x)$ and received at receiver $x_r$ are identical to the waves generated at location $x_s$ with velocity model $v(x + \delta x_s)$ and received at $x_r - \delta x_s$, thus considering the wavefield $q(x, \omega)$ calculated by

$$-\frac{\omega^2}{v^2(x + \delta x_s)} q(x, \omega) = \nabla^2 q(x, \omega) + f(\omega) \delta(x - x_s).$$ \hspace{1cm} (A-3)

It can be related to $u_s(x, \omega)$ by

$$u_s(x, \omega) = q(x - \delta x_s, \omega) \approx q(x, \omega) - \delta x_s \cdot \nabla q(x, \omega) \approx q(x, \omega) - \delta x_s \cdot \nabla u(x, \omega).$$ \hspace{1cm} (A-4)

Hence, we have

$$q(x, \omega) = u_s(x, \omega) + \delta x_s \cdot \nabla u(x, \omega) \approx u(x, \omega)$$

$$+ \delta u(x, \omega) + \delta x_s \cdot \nabla u(x, \omega).$$ \hspace{1cm} (A-5)

Inserting $v(x + \delta x_s) \approx v(x) + \delta x_s \cdot \nabla v(x)$ in to equation A-3 and using Taylor expansion, we obtain

$$-\frac{\omega^2}{v^2(x)} q(x, \omega) = \nabla^2 q(x, \omega) + f(\omega) \delta(x - x_s)$$

$$- \frac{2\omega^2 \delta x_s \cdot \nabla v(x)}{v^3(x)} q(x, \omega).$$ \hspace{1cm} (A-6)

Substitution of equation A-5 into A-6 and then subtracting with equation A-1, we obtain the perturbation of the wavefield $\delta u(x)$

$$-\frac{\omega^2}{v^2(x)} \delta u(x, \omega) = \nabla^2 \delta u(x, \omega) + \delta x_s \cdot \nabla \delta u(x, \omega)$$

$$+ \left[ \nabla^2 u(x, \omega) + \frac{\omega^2}{v^2(x)} \nabla u(x, \omega) - 2\omega^2 \frac{\delta v(x)}{v^3(x)} u(x, \omega) \right].$$ \hspace{1cm} (A-7)

It can also be expressed using sensitivity kernel

$$\delta u(x, x_r, \omega) = \int \delta x_s \cdot K(x, \omega, x_r, x_s) dx,$$ \hspace{1cm} (A-8)

where

$$K(x, \omega, x_r, x_s) = \frac{\partial u(x_r, x_s, \omega)}{\partial x_s}$$

$$= K_1(x, \omega, x_r, x_s) + K_2(x, \omega, x_r, x_s),$$ \hspace{1cm} (A-9)

and

$$K_1(x, \omega, x_r, x_s) = \left[ \nabla^2 u(x, \omega) + \frac{\omega^2}{v^2(x)} \nabla u(x, \omega) \right] \times G(x_r, x, \omega).$$ \hspace{1cm} (A-10)

$$K_2(x, \omega, x_r, x_s) = -\frac{2\omega^2 \delta v(x)}{v^3(x)} u(x, \omega) G(x_r, x, \omega),$$ \hspace{1cm} (A-11)

where $K_1$ is for the spatial variation of the wavefield and it accounts for the geometric perturbation of the source position, whereas $K_2$ mainly accounts for the effects for the spatial variation of the velocity around the source. For MVA, the velocity is assumed to be smooth, we can safely omit the part of $K_2$ in the demigration process.

Further considering the term in the parentheses for $K_1$, it can be approximated as

$$\nabla^2 u(x, \omega) + \frac{\omega^2}{v^2(x)} \nabla u(x, \omega) \approx -f(\omega) \nabla \delta(x - x_s).$$ \hspace{1cm} (A-12)

The resulted wavefield perturbation can be calculated as

$$\delta u(x, x_r, \omega) = -\int \delta x_s \cdot f(\omega) \nabla \delta(x - x_s) G(x_r, x, \omega) dx.$$ \hspace{1cm} (A-13)

Using the shifting property of the Dirac delta function, we have

$$\delta u(x, x_r, \omega) = f(\omega) \delta x_s \cdot \nabla_x G(x_r, x, \omega).$$ \hspace{1cm} (A-14)
APPENDIX B
WAVEFIELD PERTURBATION FOR VELOCITY PERTURBATION

This appendix is supplement for explanation of the velocity sensitivity term of $K_v$ in equation 7. By Born approximation, we rewrite the second equation of equation 5 as

$$
\delta d_v(x, s, \omega) = \int \delta v(x, h, \omega, s, s, \omega) dx dh K_v(x, h, \omega, s) dx dh 
$$

$$
= \int \delta v(x, h, \omega, s, s, \omega) dx dh \left[ \frac{-\omega^2 G(x - h, s, \omega)}{v^3(x)} I(x, h) G(x, x + h, \omega) 
\right. 
+ G(x - h, s, \omega) I(x, h) \left. \frac{-\omega^2 G(x, x + h, \omega)}{v^3(x)} \right] \delta d_v(x, s, \omega). 
$$

(B-1)

By adjoint analysis, we can obtain the gradient for the velocity perturbation given the residual data:

$$
\delta \bar{v}(x) = \int \delta d_v(x, s, \omega) dh K_v^*(h, \omega, s, s, \omega) 
$$

$$
= \int \delta d_v(x, s, \omega) dh \left[ \frac{-\omega^2 G^*(x - h, s, \omega)}{v^3(x)} I(x, h) G^*(x, x + h, \omega) 
\right. 
+ G^*(x - h, s, \omega) I(x, h) \left. \frac{-\omega^2 G^*(x, x + h, \omega)}{v^3(x)} \right] \delta d_v(x, s, \omega). 
$$

(B-2)

Using the reciprocity of the Green’s function $G(a, b, \omega) = G^*(b, a, \omega)$, we can reformulate equation B-2 as

$$
\delta \bar{v}(x) = \int \delta d_v(x, s, \omega) dh K_v^*(h, \omega, s, s, \omega) 
$$

$$
= \int \delta d_v(x, s, \omega) dh \left[ \frac{-\omega^2 G^*(x - h, s, \omega)}{v^3(x)} I(x, h) G(x + h, x, \omega) 
\right. 
+ G^*(x - h, s, \omega) I(x, h) \left. \frac{-\omega^2 G(x + h, x, \omega)}{v^3(x)} \right] \delta d_v(x, s, \omega). 
$$

(B-3)

With equations B-1 and B-3, we can evaluate the sensitivity kernel $K_v$ and its adjoint $K_v^*$.

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