A recipe for practical full-waveform inversion in orthorhombic anisotropy

Tariq Alkhalifah¹, Nabil Masmoudi¹, and Ju-Won Oh¹

Abstract

Multiparameter full-waveform inversion (FWI) usually suffers from the inherent tradeoff in the multiparameter nature of the model space. In orthorhombic anisotropy, such tradeoff is magnified by the large number of parameters involved in representing the elastic or the acoustic approximation of such a medium. However, using a new parameterization with distinctive scattering features, we can condition FWI to invert for the parameters to which the data are sensitive at different stages, scales, and locations in the model. Specifically, with a combination made up of a velocity and particular ratios of the elastic coefficients, the scattering potential and the anisotropy parameters has stationary scattering radiation patterns as a function of the type of anisotropy. With our new parameterization, P-wave data are mainly sensitive to the scattering potential of four parameters: the horizontal velocity in the x₁ direction, v₁; ε₁, which provides scattering mainly near zero offset in the x₁-x₂ vertical plane; ε₂, which is the ratio of the horizontal velocity squared in the x₁ and x₂ directions; and δ₁ describing the anellipticity in the horizontal plane. Since, with this parameterization, the radiation pattern for the horizontal velocity and ε is azimuthally independent, we can perform an initial VTI inversion for these two parameters, and then use the other two parameters to fit the azimuthal variation in the data. This can be done at the reservoir level or any region of the model. Including the transmission from reflections, the migration velocity analysis (MVA) component, into the picture, multiazimuth surface seismic data are mainly sensitive to the long-wavelength components of v₁, two dimensionless parameters through the diving waves, and three other dimensionless parameters in the transmission to or from reflectors (especially, in the presence of large offsets). They are also sensitive to the short-wavelength component of v₁ and ε.

Introduction

Orthorhombic anisotropy has emerged as the most practical model that combines the natural, mostly vertical, layering of the earth (due to gravity) and the normal horizontal stress variations. Since P-wave data are the easiest to acquire and have the highest quality, the acoustic assumption can reduce the complexity of the inversion considerably (Alkhalifah, 2003; Song and Alkhalifah, 2013). However, in FWI, the elastic story is useful if we are heading toward higher resolution (Tarantola, 1986) where amplitudes matter.

In the acoustic case, six parameters that may change laterally or vertically govern the orthorhombic model description (Tsankin, 1997; Alkhalifah, 2003). This is five more than we are used to seeing in the isotropic case (P-wave velocity). Even with multiazimuth data, the problem is complicated by the complex tradeoff in the data sensitivity to the parameters. In addition, if the azimuth of the orthorhombic axis is unknown, we will need to invert for it as well. As a result, we have a potential for a large null space in the inversion problem. However, we have realized recently that these parameters can be inverted at different scales within the surface P-wave seismic data wavelength range (Alkhalifah, 2016; Masmoudi and Alkhalifah, 2016). In FWI, we usually have a reflectivity scale and a traveltime resolution (wave propagation) scale. The data are sensitive to these parameters (other than velocity) at mainly one resolution scale; thus, at that scale we can constrain the parameter. Some of these parameters may provide reservoir-scale information with higher-frequency data (Masmoudi et al., 2016).

In any multiparameter inversion, like in anisotropic materials, the goal is to choose a parameterization that can provide a minimum set of parameters that the data are sensitive to with minimal tradeoff (Burridge et al., 1998; Plessix and Cao, 2011; Prieux et al., 2011; Operto et al., 2013). Such parameterization depends on data acquisition, and, in the case of surface seismic data, it depends on the available azimuths and offsets. Finding a minimum set of parameters that can explain the data can lead to a better inversion. Alkhalifah and Plessix (2014) analytically analyzed the radial dependency (radiation pattern) of the anisotropic parameter perturbation in acoustic transversely isotropic media with a vertical symmetry axis (VTI). They advocated using certain combinations of parameters for various FWI strategies, including those that start with a model obtained from MVA and those obtained from inverting diving-wave energy. Alkhalifah (2016) analyzed the data sensitivity to the long- and short-wavelength components of the model in VTI media. As a result of the analysis, Alkhalifah (2016) suggested that the combination that includes the NMO velocity, v₁, and the anellipticity parameter, η₁, was the most practical for the long-wavelength model components, extracted mainly from velocity analysis or tomography. On the other hand, a combination that includes v₁ and ε₁, focused on the...
short-wavelength components extracted from FWI, has the ability to further update the long-wavelength features of the horizontal velocity using the diving waves. In this case, and based on the radiation patterns, high-resolution $\eta$ is not reasonably resolvable and, thus, is kept constant through the FWI process.

The sensitivity of the data to perturbations in the model parameters is well represented by the first (linear) term of the Born series, which is adjointly related to the gradient of FWI. However, data are also very sensitive to the transmission components of these model parameters to and from predicted perturbations and are well approximated by the long-wavelength part of the second term of the Born series. This nonlinear component of the influence usually is handled under the context of migration velocity analysis (MVA) in the image domain (Yilmaz and Chambers, 1984) or, recently, by reflection waveform inversion (RWI) in the data domain (Xu et al., 2012). Here, we will focus on analyzing the scattering and transmission potential behavior of the model parameters using a new parameterization for orthorhombic anisotropy. In this case, we analyze inversion gradients corresponding to classical scattering and those corresponding to transmission from a predicted reflector associated with RWI or MVA.

A new parameterization

Masmoudi and Alkhalifah (2016) analyzed the radiation patterns of the scattering potential for many parameterizations for an orthorhombic assumption of the medium and introduced a new parameterization based on deviation parameters with potentially attractive features for FWI and MVA. We will start by reviewing this new parameterization and its corresponding radiation patterns. We will look at its transmission and scattering features.

Guided by the insights presented in Alkhalifah and Plessix (2014), we concluded that a parameterization given by one velocity (or two in the elastic case) and dimensionless parameters allows for continuity of the scattering potential of key parameters as we move from higher symmetry anisotropy to lower ones. For the acoustic case, the key parameter in the suggested parameterization is the horizontal, $v_h = v_s$ (or for RWI, the NMO, $v_n = v_s$) velocity in the $x_3$-$x_1$ (or $x_3$-$x_2$) plane, where $v_s = v_x(1 + 2\varepsilon)$ and $v_h = v_x(1 + 2\delta)$, with $\delta = \delta_3$, and $\varepsilon = \varepsilon_1$ defined in the same plane. Here, $v_s$ is the vertical P-wave velocity. The subscript number convention used here differs from Tsvankin (1997), who used the normal to the plane to define the parameters, and this definition is inline with Alkhalifah (2003). Thus, we remove the subscript to mitigate the confusion. Alkhalifah (2016) suggested that the $v_s$, $\eta$, and $\varepsilon$ combination was optimal for FWI, and the $v_n$, $\eta$, and $\delta$ combination was optimal for MVA in acoustic VTI media. In both of these parameterizations that include a velocity and two dimensionless parameters, the scattering features of the velocity are the same as that for isotropic materials (not changing radially), which allowed for a continuity between an isotropic model of the medium and a VTI one. Accordingly, with similar parameterization, we can utilize orthorhombic parameters that maintain the VTI scattering features regardless of azimuth.

In the acoustic orthorhombic case, we will need three parameters in addition to the VTI parameters. Thus, defining the parameters with respect to one of the vertical symmetry planes (deviation parameters) allows the scattering from the VTI parameters to be azimuth independent (Masmoudi and Alkhalifah, 2016). Along with $\delta_3$, which is the $\delta$ parameter defined from the $x$-axis along the TI horizontal plane, we can utilize two of

$$
\delta_\parallel = \frac{1}{2} \left( \frac{v_{s2}^2}{v_{s1}^2} - 1 \right),
\delta_\perp = \frac{1}{2} \left( \frac{v_{h2}^2}{v_{h1}^2} - 1 \right),
\eta_\parallel = \frac{\varepsilon_3 - \delta_\parallel}{1 + 2\delta_\parallel},
\eta_\perp = \frac{\varepsilon_1 - \delta_\perp}{1 + 2\delta_\perp},
$$

where $v_{s2}$ and $v_{h2}$ are the horizontal and NMO velocities in the $x_2$-$x_1$ ($y$-$z$) plane. The choice of the two deviation parameters depends on the choice of velocity. For the $v_s$ combination, we use $\eta_\parallel$ and $\varepsilon_3$, and for the $v_n$ combination, we use $\delta_\parallel$ and $\varepsilon_1$ for reasons given in Masmoudi and Alkhalifah (2016) and described below.

The parameters $\varepsilon_\parallel$ and $\eta_\parallel$, as well as $\delta_\parallel$, describe the deviation from VTI; if we set them to zero, the medium reduces to VTI. Using this parameterization, Masmoudi and Alkhalifah (2016) derived radiation patterns for acoustic orthorhombic media. We analyze those radiation patterns next and obtain some insights for inversion.

The scattering potential of the parameters (FWI)

For acquisition on the earth surface, the linearized data response to parameter perturbations is at the heart of predicting such perturbations, and thus, updating the velocity model in waveform-inversion applications. In this section, we focus on the scattering potential from a horizontal reflector. This scattering potential is defined as a function of the scattering angle, $\theta$, the angle between the background (source) and the scattered (receiver) wave paths measured at the scattering point. A scattering angle of zero implies mainly zero-offset reflection data, while $\theta = 180^\circ$ implies direct or diving waves. For our 3D orthorhombic case, the scattering potential (radiation pattern) is also dependent on the azimuth of the incident and scattered wave paths at the scattering point. The azimuth is measured from the $x_1$-$x_3$ (or $x$-$z$) symmetry plane of the orthorhombic medium.

The resulting resolution of the perturbation extracted from the data depends on the scattering angle. Thereafter, we consider an isotropic background model. Since we are dealing with scattering, the effect of the background medium on the perturbation scattering potential is small. The resolvability of the desired scattering object is provided by diffraction tomography principles in which the model wavenumber vector magnitude is proportional to the frequency, $\omega$, but also proportional to the cosine of half of the scattering angle, $\theta$ (Miller et al., 1987; Jin et al., 1992; Thierry et al., 1999), given by:

$$
|k_n| = \frac{\omega}{v_0} \cos \frac{\theta}{2},
$$

where $v_0$ is the velocity at the scattering point of the background isotropic model. As a result, we expect data recorded at large
reflection scattering angles to produce lower resolution. Keeping this relation in mind, we analyze the radiation patterns shown in Figures 1a–1d.

As mentioned earlier, the scattering potential of the velocity parameter is isotropic over scattering angles and azimuths (Figure 1). Both \( \eta \) and \( \epsilon \) produce scattering at certain polar angles (\( \theta \)); the scattering is stationary with azimuth. As suggested by Alkhalifah (2016), for surface seismic data, \( \eta \) has low scattering energy focused mainly at the far offsets. It probably can be ignored at the acoustic stage. Meanwhile, \( \epsilon \) admits scattering at small offsets and can be used to match the amplitudes to overcome the limitations of the acoustic assumption. As we deviate from zero azimuth (Figures 1b and 1c), the orthorhombic parameters, \( \epsilon_d \), \( \eta_d \), and \( \delta_3 \) start to induce scattering. While the \( \delta_3 \) influence resides in the middle azimuths (around \( \phi = 45^\circ \)), the other two parameters have their maximum influence at \( \phi = 90^\circ \). Again, the \( \eta_d \) influence resizes mainly at large offsets and might be ignored at the acoustic stage. Meanwhile, \( \epsilon_d \) addresses the variation in the horizontal velocity mainly for diving waves in the \( \phi = 90^\circ \) direction. In the elastic case, where we usually focus on amplitudes and with reasonable offsets in the acquired data, \( \eta \) and \( \eta_d \) could play a role in describing the scattering variation away from vertical.

Since the maximum perturbation in the deviation parameters resides at \( \phi = 90^\circ \), we can use this fact to search for the azimuth of the vertical symmetry plane. With wide- or full-azimuth data, we scan over orthogonal azimuths to find the pair that admits the largest \( \epsilon_d \) gradient energy. Theoretically, this azimuth combination represents the symmetry plane of the orthorhombic model. However, since \( \epsilon_d \) induces scattering at larger angles (Figure 1c), the expected resolution of the inverted azimuth direction is low. This holds for the acoustic assumption.

The transmission component from a reflection (MVA)

The long-wavelength component of the velocity model — or the anisotropy parameters — typically controls the majority of wave propagation characteristics affecting the shape of the waves, which is conveniently described, in the high frequency asymptotic, by traveltimes. To analyze the behavior of this component for reflections, we focus on the transmission angles of the scattering (\( \theta = 180^\circ \)) and from a horizontal reflector. However, the depth of this reflector also depends on the background velocity. Since the background medium is assumed isotropic, we scale the vertical axis by the background velocity, which is assumed equal to the vertical velocity since we are dealing with horizontal reflectors (Alkhalifah et al., 2001; Plessix, 2013). This step will allow us to obtain radiation patterns that reflect the earth-surface nature of our acquisition, in which the trade-off between velocity and reflector depth is considered, and for anisotropic media, it is simply unresolvable.

Thus, we mitigate the effect of the reflector’s depth by mapping the vertical axis, \( z \), to vertical time using the relation \( dz = v_z dt \). The resulting wave equation for orthorhombic anisotropy becomes independent of the vertical velocity (and \( \delta \)). As expected, the transmission from the earth surface in the vertical direction to a horizontal reflector (zero offset) does not include long-wavelength information. Only multioffset data, as is generally known, allow for resolution of velocity, and in orthorhombic media that velocity is \( v_z \) in the \( x \) direction, with \( \delta_3 \) describing the NMO velocity deviation in the \( y \) direction. Larger offsets should illuminate long wavelength \( \eta \) in the \( x \) direction and \( \epsilon_d \), which represents the deviation in the nonhyperbolic moveout, in the \( y \) direction, while \( \delta_3 \) requires large offsets in the intermediate azimuths.

These intuitive statements are reflected in the transmission radiation patterns for the \( v_z \) parameter combination corresponding to scaling the vertical axis to vertical time shown in Figures 2a–2c. Since these radiation patterns are extracted from the Born linearized data sensitivity to perturbations, these particular radiation patterns highlight the MVA, including RWI, resolvability of the orthorhombic parameters. As expected, when transmission occurs vertically (from horizontal reflectors), the data are insensitive to all the parameters. This is courtesy of the velocity-depth ambiguity in that direction. In fact, part of the requirement for a successful RWI free of reflectivity imprint is that we match the data perfectly at zero offset in the demigration step. Thus, the residual for the zero-offset path is zero, and the resulting gradient for that path has generally zero energy. In the zero-azimuth (\( \alpha \)) direction (Figure 2a), the influence of
\[ \psi = \theta - 1 - \phi \]

The interesting feature of defining the elastic additions by \( \psi \), \( \gamma \), and \( \psi_n \) is that we maintain the radiation patterns we ended up with for P-waves in the acoustic case (the linearized scattering potential). Figure 3a shows the P-wave response to perturbations in three elastic parameters. While, as expected, a \( \psi \) perturbation induces scattering in the mid-angle range, like what we encounter in isotropic media (Taranolta, 1986), as such, no \( \psi \) information can be extracted from transmission P-waves, \( \gamma \) perturbations do not induce P-wave scattering at any angle, we can simply ignore it. On the other hand, \( \gamma_n \) induces scattering at azimuths larger than zero culminating in a scattering potential overlapping the \( \psi \) one at 90° azimuth.

Adding these three parameters to the inversion of P-waves (like in marine data) only adds more potential for null space. Using ocean-bottom-cable (OBC) data allows us to record multicomponent data, which should better constrain the elastic part of the parameter set. Ironically, as we observe from Figure 3b and from Oh and Alkhalifah (2016), \( \gamma \) produces no scattering of incident P-waves. Thus, \( \gamma \) induces scattering only when the incident wave is a shear one. As a result, we can ignore this parameter. On the other hand, a \( \psi \) perturbation excites stationary scattering of SV (or S1) waves with azimuth; however, it does not excite SH (or S2) waves (Oh and Alkhalifah, 2016). We use
thought of as perturbations themselves. This opens the door for

From the fact that dimensionless anisotropic parameters can be

We may have a large enough wavelength to accommodate the

Anisotropy parameters as perturbations

Though we cannot constrain the low wavenumber components

the terms SV and SH (instead of S1 and S2) to reflect our or-

The wavefield response to perturbations in the elastic pa-

The vertical-component shot gathers represented in Figure 5

Application on North Sea data

The Volve data set corresponds to a 3D OBC acquisition with

Figure 4. The wavefield response to 13 scatterers placed at an equal distance from the source in the middle (red dot). The scatterers correspond to (a) \(v_s\), (b) \(\gamma\), and (c) \(\gamma_d\) perturbations. The locations of these scatterers are meant to provide a good coverage of angles.

Figure 5. Example of vertical-component shot gathers from three different cables.
to data corresponding to vertical displacements as observed data. Also, for cost purposes, we invert for data with frequencies ranging between 2.75 Hz to 5 Hz. The initial models are shown in Figures 6a–6d. They were obtained by smoothing VTI tomographic results provided by the data owners. The horizontal P-wave (Figure 6a) and S-wave (Figure 6b) velocities reflect a general velocity increase with depth with a mild low-velocity zone under 3.0 km depth in the critical reservoir region (Szydlík et al., 2007). The $\eta$ model, not shown here, is extracted from the provided $\delta$ and $\varepsilon$, but not updated here. The initial $\varepsilon$ model, shown in Figure 6c, conveys the long-wavelength behavior of the anisotropy, as we expect FWI to focus on reflectivity updates for $\varepsilon$ (Alkhalifah, 2016). Meanwhile, the $\varepsilon_d$ model (Figure 6d) is set to zero, as azimuthal anisotropy information is absent.

As a first stage of the 3D elastic ORT FWI, we perform 35 iterations to update only $v_h$ and $v_s$ to recover an isotropic elastic model. The resulting inverted models are shown in Figures 7a–7b. Now, as a result, we see a clear low-velocity zone at 3.1 km depth. In the depth slice, the boundaries of this zone are also better defined.

In the second stage, we include $\varepsilon$ in the inversion (15 iterations), which scatters energy mainly at narrow opening angles; thus we can mainly obtain high-resolution updates resulting in the model shown in Figure 7c. The high-resolution features are helpful in fitting the reflection amplitude at near offsets, where $v_h$ and $v_s$ are not enough in the orthorhombic representation of the model. In addition, the inverted $\varepsilon$ reveals a low-anisotropy zone at 3.0 km depth, with area given in the depth slice that correlates with the low-velocity zone. In the final stage, we include $\varepsilon_d$ in the inversion (10 iterations), which should reveal the azimuthal anisotropy with low resolution at depth as the radiation pattern predicts. Figure 7d supports such expectations with generally low regional azimuthal anisotropy. However, the small offset in the crossline direction with a maximum of 1 km limited our ability to resolve $\varepsilon_d$, especially at depth. Using velocities extracted from a slightly deviated well located at about $[8.2$ km, $3$ km] as given by the color dots in Figure 7a showing the location of the top and bottom of the well, we check the accuracy of the inverted vertical P-wave velocity at the location of the well. Also included in the graph are the initial velocity and the provided VTI-based tomographic velocity. Clearly, the FWI velocity for data up to 5 Hz provided higher-resolution information with better fit to the well than the tomographic result, especially at depth. The vertical velocity, however, is affected by the inverted $\varepsilon$, as, theoretically, we cannot constrain the long-wavelength vertical velocity in an orthorhombic (or VTI) model. As a result, the vertical velocity from the inverted model is generally lower than the well velocity. However, it reasonably captured many of the higher-resolution features.

Figure 6. The initial 3D models for (a) $v_h$, (b) $v_s$, (c) $\varepsilon$, and (d) $\varepsilon_d$. The plots show three slices of the 3D model: a horizontal slice (top), an inline slice (bottom left), and a crossline slice (bottom right). The location of slices are given by the blue lines.
Conclusions

Through the analysis of the data dependency on the scattering and transmission components of a particular (we think optimal) anisotropic parameter representation of orthorhombic media, we managed to understand our inversion limitations and outline a strategy for a practical inversion of orthorhombic anisotropy using multiazimuth surface seismic data acquisition. The combinations we used here, given by either the horizontal or NMO velocity and the corresponding dimensionless parameters, provided minimum tradeoff for FWI and MVA methods, respectively. Handling the azimuth variation as a deviation from VTI allowed the VTI parameters to have stationary scattering potential with azimuth, and thus, decouple the azimuth dependency to the new deviation parameters. A similar approach is utilized in the elastic stage, allowing for the application of the elastic orthorhombic inversion in a separate stage.

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Figure 7. The inverted 3D models for (a) $v_h$, (b) $v_s$, (c) $\epsilon$, and (d) $\epsilon_d$. The plots show three slices of the 3D model: a horizontal slice (top), an inline slice (bottom left), and a crossline slice (bottom right). The location of slices are given by the blue lines. The blue and red dots in (a) show, respectively, the locations of the top and bottom of the well used in Figure 8.

Figure 8. A vertical profile of the models extracted from the well, which starts from the red dot on the top and ends at the blue dot at the bottom in Figure 7a.
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Corresponding author: tariq.alhalifah@kaust.edu.sa

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