Mapping moveout approximations in TI media

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ABSTRACT

Moveout approximations play a very important role in seismic modeling, inversion, and scanning for parameters in complex media. We developed a scheme to map one-way moveout approximations for transversely isotropic media with a vertical axis of symmetry (VTI), which is widely available, to the tilted case (TTI) by introducing the effective tilt angle. As a result, we obtained highly accurate TTI moveout equations analogous with their VTI counterparts. Our analysis showed that the most accurate approximation is obtained from the mapping of generalized approximation. The new moveout approximations allow for, as the examples demonstrate, accurate description of moveout in the TTI case even for vertical heterogeneity. The proposed moveout approximations can be easily used for inversion in a layered TTI medium because the parameters of these approximations explicitly depend on corresponding effective parameters in a layered VTI medium.

INTRODUCTION

The theory of moveout approximations in anisotropic media with a tilted symmetry axis (TTI) has been developed in recent years (Sena, 1991; Pech et al., 2003; Zhou et al., 2004; Grechka and Pech, 2006; Huang et al., 2008). Generally, the presence of tilt results in significant complications in moveout expressions and traveltime parameters. Golikov and Stovas (2012) derive the Taylor series for traveltime squared in a homogeneous TTI medium with horizontal interfaces and show the difference between one-way and two-way traveltimes. This Taylor series is defined at the apex position and, generally, is not suitable for zero offset. To derive the moveout approximation in a TTI medium, Alkhalifah (2011) proposes a perturbation method using the tilt angle and the anellipticity parameter η as the perturbation parameters. Stovas and Alkhalifah (2012b) extend this method by perturbing η only, which allows the use of arbitrary tilt. They derive a trial solution of the TTI eikonal equation, and apply the Shanks transform to stabilize the truncated power series.

In this paper, we use the moveout mapping concept proposed earlier by the authors (Stovas and Alkhalifah, 2012a) to derive the TTI moveout approximations from the ones defined for a VTI medium. By applying the mapping transformation, we convert a few well-known moveout approximations from VTI to TTI media and test their accuracy. We show that the mapped moveout approximations result in approximate traveltime values defined at zero and infinite offset. The derived approximations are tested on synthetic examples. We also discuss an extension of our approximations to two-way traveltime moveout, and their applicability for multilayered TTI models.

THE MOVEOUT MAPPING IN TI MEDIA

The parametric equations for offset x and one-way traveltime t for a homogeneous VTI medium are given by

\[ x(p) = \frac{p v_{nmo}^4 t_0}{(1 - 2\eta p^2 v_{nmo}^2)^{3/2} \sqrt{1 - (1 + 2\eta)p^2 v_{nmo}^2}}, \]

\[ t(p) = t_0 \frac{[1 - 2\eta p^2 v_{nmo}^2]^2 + 2\eta p^4 v_{nmo}^4}{(1 - 2\eta p^2 v_{nmo}^2)^{3/2} \sqrt{1 - (1 + 2\eta)p^2 v_{nmo}^2}}, \]

where \( t_0 \) is the zero-offset one-way traveltime, \( v_{nmo} = v_0 \sqrt{1 + 2\delta} \) is the normal moveout velocity, \( \eta = (\varepsilon - \delta)/(1 + 2\delta) \) is the anelliptical anisotropy parameter, \( v_0 \) is the vertical velocity, \( \varepsilon \) and \( \delta \) are the Thomsen (1986) anisotropy parameters, and \( p \) is the horizontal projection of the slowness vector, i.e., the ray parameter.

In an homogeneous 2D TI medium with horizontal interface, the mapping of the moveout from the VTI to the TTI model is given by the following rational equations (Stovas and Alkhalifah, 2012):
\[ x_n(p) = z \frac{x(p) \cos \theta + z \sin \theta}{z \cos \theta - x(p) \sin \theta}, \]
\[ t_n(p) = t(p) \frac{z}{z \cos \theta - x(p) \sin \theta}. \]  

(2)

where \((x, t)\) and \((x_n, t_n)\) are the offset-traveltime points in a VTI and a TTI model, respectively, \(p\) is the horizontal slowness in a VTI model, \(z\) is the layer thickness, and \(\theta\) is the tilt angle. We can select a few specific points to show the change in their position due to change in the direction of the symmetry axis, \((x=0, t=t_0) \Leftrightarrow (x=\pm z \tan \theta, t=t_0 / \cos \theta)\) and \((x=-z \tan \theta, t=t_0 / \cos \theta) \Leftrightarrow (x=0, t=t_0)\), where \(t_0\) is the zero-offset one-way traveltime in a TTI medium.

The inverse transform is defined as

\[ x(p) = z \frac{x_n(p) \cos \theta - z \sin \theta}{z \cos \theta + x_n(p) \sin \theta}, \]
\[ t(p) = t_n(p) \frac{z}{z \cos \theta + x_n(p) \sin \theta}. \]  

(3)

The function \(t_1(x_n)\) is no longer symmetric and has a minimum at \(x_n = x_{0n}\), which is defined by equation \(dt_1/dx_n = 0\). The latter one has a solution, \(p_n = p \cos \theta + q \sin \theta = 0\) (the zero horizontal slowness in \(p_n, q_n\) space). In Figure 1, we show the traveltime functions computed for a homogeneous TTI medium. We consider a layer of thickness \(z = 1\) km, the symmetry axis velocity \(v_0 = 2\) km/s, the symmetry axis normal moveout velocity \(v_{uno} = 2.2\) km/s, anelliptic parameter \(\eta = 0.2\), and three different tilt angles: \(\theta = 0\) (VTI case), \(\theta = 0.2\), and \(\theta = 0.4\). We can see the different behavior for different \(t_1(x_n)\) at \(x_n = 0\). The TTI moveout function is more complicated than the VTI one. The apex is shifted to negative or positive offset depending on the sign of the tilt angle with respect to the vertical axis, and traveltime becomes nonsymmetric with respect to the minimum position \(x_{0n}\) (Golikov and Stovas, 2012).

**MAPPPING OF MOVEOUT APPROXIMATIONS**

There are many different moveout approximations developed for isotropic and VTI media (Malovichko, 1978; Alkhalifah and Tsvankin, 1995; Fomel and Stovas, 2010). Deriving a moveout approximation for TTI media is a nontrivial task. Alkhalifah (2011a) proposes the moveout approximations based on the linearization of eikonal equation in \(\eta\) only, which is valid for arbitrary values of tilt.

In this paper, we introduce another approach to map the moveout approximation from a VTI to a TTI medium based on the exact mapping equations. The advantage of this approach is that the accuracy of the mapped moveout approximation is exactly the same as for original moveout approximation developed for a VTI medium. We start with an approximation for VTI media in the form \(t(x)\) with the following parameters: the zero-offset traveltime \(t_0\), the normal moveout velocity \(v_n\), and the anelliptic parameter \(\eta\). Using equation 3, we obtain a general approach to transform any moveout function from VTI to TTI medium by using the following formula:

\[ t_n(x_n) = t(x = x_n \cos \theta - z \sin \theta) \frac{\cos \theta + \frac{x_n}{z} \sin \theta}{\cos \theta + x_n \sin \theta}. \]  

(4)

Let us illustrate transformation 4 with a few known moveout approximations. In all the following equations, we will use \(z = v_0 t_0\).

The truncated Taylor series for traveltime squared in a homogeneous VTI medium is given by (Hake et al., 1984)

\[ t^2(x) = t_0^2 + \frac{x^2}{v_n^2} - \frac{2\eta x^4}{v_n^4} + \ldots. \]  

(5)

The direct application of transformation 4 for the truncated series 5 gives

\[ t_n^2(x_n) = t_0^2 \left[ (v_n^2 \cos^2 \theta + v_0^2 \sin^2 \theta) + 2v_0 v_n (v_n^2 - v_0^2) \sin 2\theta + x_n^2 (v_n^2 \sin^2 \theta + v_0^2 \cos^2 \theta) \right] \]
\[ \quad - 2\eta \left[ t_0^2 (v_n/v_0 \sin \theta - x_n \cos \theta) \right] \]
\[ \quad + \left[ t_1^2 - 2\eta t_1^2/t_1^2 \right]. \]  

(6)

where

\[ t_1 = \frac{(x_n \sin \theta + v_0 t_0 \cos \theta)}{v_0}, \]
\[ t_2 = \frac{(x_n \cos \theta - v_0 t_0 \sin \theta)}{v_n}. \]  

(7)

The first term in equation 6 corresponds to the hyperbolic part of the moveout, while the second term represents the nonhyperbolic part. By expanding equation 6 in Taylor series with respect to \(x_n\), we obtain

\[ t_n^2(x_n) = t_0^2 \cos^2 \theta \left[ 1 + t_0^2 \tan^2 \theta - \frac{2\eta t_0^2}{v_n^2} \tan^4 \theta \right] \]
\[ + \frac{v_0 v_n (v_n^2 - 1)}{v_0^2} \sin 2\theta + \frac{2\eta t_0^2}{v_n^2} (3 + \cos 2\theta) \tan^3 \theta \]
\[ + \frac{x_n^2 \cos^2 \theta}{v_n^2} \left[ 1 + \tan^2 \theta \left( \frac{v_n^2}{v_0^2} - 2\frac{v_0 v_n^2}{v_n^2} (6 + 8 \tan^2 \theta + 3 \tan^4 \theta) \right) \right] \]
\[ + \frac{8\eta v_0 \tan \theta}{v_n^2} \left( \frac{v_n^2}{v_0^2} - 2\frac{v_0 v_n^2}{v_n^2} (6 + 8 \tan^2 \theta + 3 \tan^4 \theta) \right) \]
\[ + \frac{t_1^2 - 2\eta t_1^2/t_1^2}{t_1^2} \frac{v_n^2}{v_0^2} \cos^2 \theta + \ldots. \]  

(8)
Note that series 8, for TTI media, contains even and odd power terms in offset. This is because we compute the one-way moveout equation in a TTI medium (Golikov and Stovas, 2012). The odd-order terms in offset reduce to zero when \( \theta = 0 \) or \( \theta = \pi/2 \). Each Taylor-series coefficient in series 8 is approximate and related to the approximation given by the truncated Taylor series 5. If \( \eta = 0 \) (elliptical TTI medium), equation 8 reduces to a hyperbolic equation with a shifted apex (Golikov and Stovas, 2012). If \( \theta = 0 \) (VTI medium), equation 8 reduces to equation 5.

Applying the transformation 4 to the first approximation, which we consider is the hyperbolic approximation (Taner and Koehler, 1969)

\[
t^2(x) = t_0^2 + \frac{x^2}{v_n^2},
\]

(9)
yields the hyperbola with a shifted apex position (first term in equation 6),

\[
t_n^2(x_n) = t_1^2 + t_2^2 = t_0^2 + \frac{(x_n - x_{0n})^2}{v_{n,0}^2},
\]

(10)
and the new traveltime parameters are given by

\[
t_{0n}^2 = t_0^2 \frac{v_0^2}{v_0^2 \cos^2 \theta + v_n^2 \sin^2 \theta},
\]

\[
x_{0n} = \frac{v_0 v_n (v_n^2 - v_0^2) \sin \theta \cos \theta}{v_0^2 \cos^2 \theta + v_n^2 \sin^2 \theta},
\]

\[
v_{n,0}^2 = \frac{v_0^2 v_n^2}{v_0^2 \cos^2 \theta + v_n^2 \sin^2 \theta}.
\]

(11)
The moveout approximation 10 is valid for elliptical TTI medium (Golikov and Stovas, 2012). The traveltime function is a hyperbola with shifted apex.

For the second approximation, the shifted hyperbola equation (Malovichko, 1978)

\[
t(x) = t_0 \left[ 1 + \frac{1}{S} \left( \sqrt{1 + S \frac{x^2}{v_n^2 t_0^2}} - 1 \right) \right],
\]

(12)
we obtain

\[
t_n(x_n) = t_1 \left[ 1 + \frac{1}{S} \left( \sqrt{1 + S \frac{x_n^2}{t_1^2 t_0^2}} - 1 \right) \right],
\]

(13)
where \( S \) is the heterogeneity coefficient responsible for nonhyperbolicity of the traveltime function. For a homogeneous VTI medium, \( S = 1 + 8\eta \).

Applying the transformation 4 to the third approximation, which is the rational approximation proposed by Alkhalifah and Tsvankin (1995),

\[
t^2(x) = t_0^2 + \frac{x^2}{v_n^2} - \frac{2\eta x^4}{v_n^2 t_0^2 \left( 1 + (1 + 2\eta) \frac{x^2}{v_n^2 t_0^2} \right)},
\]

(14)
yields

\[
t_n^2(x_n) = t_1^2 + t_2^2 - \frac{2\eta t_1^2}{t_1^2 + (1 + (1 + 2\eta) \frac{t_2}{t_1})}.
\]

Equation 15 can be rewritten as the rational approximation

\[
t_n^2(x_n) = \frac{a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n^3 + a_4 x_n^4}{b_0 + b_1 x_n + b_2 x_n^2 + b_3 x_n^3 + b_4 x_n^4}
\]

(16)
with parameters

\[
a_0 = v_n^4 t_0^4 (v_n^4 \cos^4 \theta + 2v_n^2 v_0^2 (1 + \eta) \sin^2 \theta \cos^2 \theta + v_0^4 \sin^4 \theta),
\]

\[
a_1 = 4v_n^2 v_0^2 \cos \theta \sin \theta (v_n^4 \cos^2 \theta - v_n^2 v_0^2 (1 + \eta) \cos 2\theta - v_0^4 \sin^2 \theta),
\]

\[
a_2 = 2v_n^2 v_0^2 (3v_n^4 \sin^2 \theta \cos^2 \theta + v_n^2 v_0^2 (1 + \eta) (1 - 6 \sin^2 \theta \cos^2 \theta) + 3v_0^4 \sin^2 \theta \cos^2 \theta),
\]

\[
a_3 = 4v_0 t_0 \cos \theta \sin \theta (v_n^4 \cos^2 \theta + 2v_n^2 v_0^2 (1 + \eta) \cos 2\theta - v_0^4 \cos^2 \theta),
\]

\[
a_4 = v_n^4 \sin^4 \theta + 2v_n^2 v_0^2 (1 + \eta) \sin^2 \theta \cos^2 \theta + v_0^4 \cos^2 \theta,
\]

\[
b_0 = v_n^4 v_0^4 t_0^4 (v_n^4 \cos^2 \theta + (1 + 2\eta) v_n^2 v_0^2 \sin^2 \theta),
\]

\[
b_1 = v_n^2 v_0^2 t_0^2 \sin 2\theta (v_n^2 (1 + 2\eta) v_0^2),
\]

\[
b_2 = v_n^2 v_0^2 (v_n^2 \sin^2 \theta + (1 + 2\eta) v_0^2 \cos^2 \theta).
\]

(17)
The variables in 17 depend on all the parameters that describe a VTI medium and linear with respect to \( \eta \). The fourth approximation we consider is the generalized moveout approximation (Fomel and Stovas, 2010),

\[
t^2(x) = t_0^2 + \frac{x^2}{v_n^2} - \frac{4\eta x^4}{v_n^2 t_0^2 \left( 1 + B \frac{x^2}{v_n^2 t_0^2} + \sqrt{1 + 2B \frac{x^2}{v_n^2 t_0^2} + C} \frac{x^2}{v_n^2 t_0^2} \right)},
\]

(18)
with parameters \( B \) and \( C \) defined from the infinite offset asymptotic,

\[
B = \frac{1 + 8\eta}{1 + 2\eta}, \quad C = \frac{1}{(1 + 2\eta)^2}
\]

(19)
transformed into

\[
t_n^2(x_n) = t_1^2 + t_2^2 - \frac{4\eta t_1^2}{t_1^2 + B t_2^2 + \sqrt{t_1^4 + 2B t_1^2 t_2^2 + C t_2^4}}.
\]

(20)
There are many other analytical moveout approximations defined for a VTI medium. We can map any of these approximations from VTI to TTI media using the mapping operator 4.

**ANALYSIS OF THE APPROXIMATIONS**

All moveout approximations defined for a VTI medium have exact values for traveltime parameters: \( t_0, v_n, \) and \( \eta \) (or \( S \)), because these parameters are obtained from the first three coefficients in the Taylor series for traveltime squared. In a TTI medium, the exact values of traveltime parameters are derived only for \( x_n = x_{0n} \neq 0 \), where \( x_{0n} \) is the position of the minimum of the traveltime.
function, i.e., \( dt_n/dx_n(x_n) = 0 \) (Golikov and Stovas, 2012). The mapped moveout approach approximates the traveltime parameters at \( x_n = 0 \).

We can analyze the vertical \( V_z \) (defined at \( x_n = 0 \)), horizontal \( V_x \) (defined at \( x_n \rightarrow \infty \)), and normal moveout (NMO) \( V_{nmo} \) (defined at \( x_n = 0 \)) velocities from the mapped moveout approximations. The first and third velocities are related to the traveltime value and the curvature of traveltime function defined at zero offset, while the second velocity gives the infinite offset asymptotic traveltime corresponding velocity. Due to the symmetry of the TTI model over \( \pi/2 \), all equations for the horizontal velocity can be obtained from the equations for the vertical velocity by interchanging of sin and cosine functions. It is interesting to analyze these velocities because they are different for different mapped moveout approximations.

For convenience, we introduce the slowness projections for symmetry axis velocity and symmetry axis normal moveout velocity, 

\[
u_{0z} = \frac{\sin \theta}{v_0}, \quad \nu_{0x} = \frac{\cos \theta}{v_0},
\]

\[
u_{nz} = \frac{\sin \theta}{v_n}, \quad \nu_{nx} = \frac{\cos \theta}{v_n}.
\]

These projections are related to the auxiliary functions \( t_1(x_n) \) and \( t_2(x_n) \) defined in equation 7,

\[
t_1(x_n = 0) = u_{0z}^2, \quad \frac{d(t_1^2)}{dx_n^2}(x_n = 0) = u_{0c}^2,
\]

\[
t_2(x_n = 0) = u_{nz}^2, \quad \frac{d(t_2^2)}{dx_n^2}(x_n = 0) = u_{nc}^2.
\]

The explicit equations for \( V_z \), \( V_x \), and \( V_{nmo} \) computed from the mapped moveout approximations are given in Appendix A. The approximated vertical, horizontal, and normal moveout velocities versus the tilt angle and parameter \( \eta \) for a TTI medium with parameters, \( v_0 = 2.0 \text{ km/s} \), and \( \delta = 0.1 \) are plotted in Figures 2-4, respectively. We can see that the results for vertical, horizontal, and normal moveout velocities by the hyperbolic and the shifted hyperbola approximations are very different from the ones obtained from the rational and generalized approximations. The vertical velocity increases with increasing parameter \( \eta \) for nonzero tilt angle for all nonhyperbolic approximations. The largest vertical velocity is observed for shifted hyperbola approximation. The horizontal moveout velocity shows similar behavior with respect to a change in parameter \( \eta \), but decreases when the tilt angle increases. The normal moveout velocity (for all approximations apart from the hyperbolic one) has the maximum value for the tilt angle of 40°. Although the structure of \( V_z(\theta, \eta) \) and \( V_x(\theta, \eta) \) are similar for the rational and generalized approximations, the structure of \( V_{nmo}(\theta, \eta) \) is quite different (Figure 4).

We compute the relative errors \( (V_{\text{approx}} - V_{\text{exact}})/V_{\text{exact}} \) in the vertical and normal moveout velocity from the mapped approximations for a homogeneous TTI model with parameters, \( v_0 = 2.0 \text{ km/s} \), \( v_{nmo} = 2.2 \text{ km/s} \), and \( \eta = 0.2 \). These errors plotted versus tilt angle in Figure 5 to the top and bottom, respectively. One can see that the generalized moveout approximation gives the most accurate results, while the hyperbolic approximation gives the worst results. Generally, the accuracy for normal moveout velocity is worse than the accuracy for vertical velocity. This is due to the fact that the NMO velocity is defined from the curvature of approximated traveltime function, while the vertical velocity is related to the value of the function itself.

**NUMERICAL EXAMPLES**

To illustrate the accuracy of the proposed approximation, we select a homogeneous TTI model with parameters used in Figure 5 and \( \theta = 30^\circ \). In Figure 6, we display the relative error in traveltime obtained from selected moveout approximations being mapped from a VTI medium. The generalized approximation gives the best
results with a maximum relative error of about 0.0002 on the offset/depth spread up to five. The worst results, as expected, are obtained from the hyperbolic approximation. Note that, at offset 

\[ x_n = z \tan \theta \]

which corresponds to \( x = 0 \), all moveout approximations fit the exact traveltime perfectly. That is because these offsets correspond to on-axis position for TTI and VTI media. In Figure 7, we show the relative error in traveltime for mapped generalized approximation for different values of symmetry tilt angles. We can see that the accuracy of the generalized moveout approximation remains the same regardless of the symmetry direction.

**DISCUSSION**

**Two-way moveout approximations**

The traveltime parameters used in moveout approximations are related to Taylor-series coefficients for a vertical slowness defined at zero horizontal slowness value (Golikov and Stovas, 2012),

\[ q(p) = c_0 + c_1 p + c_2 p^2 + c_3 p^3 + c_4 p^4 + \ldots \]  

(23)

**Figure 3.** The horizontal velocities versus tilt and parameter \( \eta \) computed from the mapped moveout approximations (a) hyperbolic, (b) shifted hyperbola, (c) rational, and (d) generalized for a homogeneous TTI model with parameters \( v_0 = 2.0 \text{ km/s and } \delta = 0.1 \).

**Figure 4.** The normal moveout velocities versus tilt and parameter \( \eta \) computed from the mapped moveout approximations (a) hyperbolic, (b) shifted hyperbola, (c) rational, and (d) generalized for a homogeneous TTI model with parameters \( v_0 = 2.0 \text{ km/s and } \delta = 0.1 \).
For a VTI medium (one-way or two-way), the odd-order coefficients in equation 27 equal zero. For one-way traveltime in a TTI medium, all the coefficients in equation 27 are nonzero, and the traveltime parameters in the series for traveltime squared are defined as (Golikov and Stovas, 2012)

\[ t_n^2(x_n) = t_0^2 + \frac{(x_n - x_{n0})^2}{v_{n0}^2} + d\frac{(x_n - x_{n0})^3}{v_{n0}^4}t_0^2 + \frac{(1 - S_2)(x_n - x_{n0})^4}{4^3v_{n0}^6} \ldots \] (24)

are defined as

\[ t_0 = zc_0, \quad x_{n0} = -zc_1, \quad v_{n0}^2 = -\frac{2c_2}{c_0}, \]
\[ d = \frac{c_3}{2}\sqrt{-\frac{c_0}{2c_2^3}}, \quad S_2 = \frac{c_0(9c_2^2 - 4c_2c_4)}{2c_2^4}. \] (25)

One can easily see that the two-way traveltime parameters are given by doubling of zero-offset traveltime \( t_0 = 2t_0 \), setting \( x_{n0} = 0 \) and \( d = 0 \), setting the normal moveout velocity \( v_{n0} = v_{n0} \), and using a slightly different value for the heterogeneity coefficient \( \eta = \frac{v_c}{v_s} \). By using any velocity estimation technique, we can compute the effective parameters: \( t_0, v_{n0}, \eta, \) and \( \theta \) for each reflector. Despite the fact that the mapped rational approximation 16 has eight parameters given in equation 17, there are only four independent parameters controlling this approximation. The first three parameters can be converted into the interval parameters by using the standard Dix-type inversion for VTI media. The effective tilt computed for the first reflector is an interval tilt value for the first layer. To compute the interval tilt values for other layers, we need to compute the effective TTI traveltime parameters:

\[ \hat{S}_2 = S_2 + 36d^2. \] (27)

For example, the rational approximation 16 expands to series 24 at \( x_n = x_{n0} \), where \( x_{n0} \) can be found from setting \( dx_n/ds_n = 0 \). After computing two-way traveltime parameters, we can define the rational approximation exactly in the same form as given in equation 14 but with the other traveltime parameters described above.

**Multilayered medium**

The one-way approximations 10, 13, 16, and 20 can easily be extended for multilayered media by using effective parameters \( t_0, v_{n0}, \eta, \) and tilt angle. Note that the first three parameters correspond to an effective VTI medium, while a tilt angle represents effective tilt in a layered TTI medium. Let us illustrate that on the rational approximation 16. By using any velocity estimation technique, we can compute the effective parameters:

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**Multilayered medium**

The one-way approximations 10, 13, 16, and 20 can easily be extended for multilayered media by using effective parameters \( t_0, v_{n0}, \eta, \) and tilt angle. Note that the first three parameters correspond to an effective VTI medium, while a tilt angle represents effective tilt in a layered TTI medium. Let us illustrate that on the rational approximation 16. By using any velocity estimation technique, we can compute the effective parameters: \( t_0, v_{n0}, \eta, \) and \( \theta \) for each reflector. Despite the fact that the mapped rational approximation 16 has eight parameters given in equation 17, there are only four independent parameters controlling this approximation. The first three parameters can be converted into the interval parameters by using the standard Dix-type inversion for VTI media. The effective tilt computed for the first reflector is an interval tilt value for the first layer. To compute the interval tilt values for other layers, we need to compute the effective TTI traveltime parameters:

\[ \hat{S}_2 = S_2 + 36d^2. \] (27)

For example, the rational approximation 16 expands to series 24 at \( x_n = x_{n0} \), where \( x_{n0} \) can be found from setting \( dx_n/ds_n = 0 \). After computing two-way traveltime parameters, we can define the rational approximation exactly in the same form as given in equation 14 but with the other traveltime parameters described above.
T_0, X_{tno}, V_{nmo}^2, D, and S_2 for layers 1 and 2, using the cumulative equations from Appendix B in Golikov and Stovas (2012), and obtain interval parameters for layer 2. The interval parameters T_0, v_{nmo} and η are obtained at the previous stage. By doing the fitting for all TTI parameters, we can estimate the interval tilt values. By repeating this procedure from top to bottom, we can estimate all interval TTI parameters.

CONCLUSIONS

We develop an analytical general 2D formula to map moveout approximations from VTI to TTI media. The method is applied on the hyperbolic, the shifted hyperbola, the rational, and the generalized approximations for a vertical symmetry axis direction. The mapped moveout approximations are also used to derive approximations for vertical, horizontal, and normal moveout velocities. The accuracy of the mapped moveout approximations is tested for an homogeneous TTI medium example. We show that the generalized moveout approximation being mapped to a TTI medium remains the most accurate one. We discuss the extension of the one-way traveltine approximations to their two-way counterparts and the applicability of proposed approximations for a layered TTI medium including the inversion into the interval model parameters. The proposed mapping procedure can be extended for 3D by introducing two apparent tilt angles.

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APPENDIX A

EQUATIONS FOR VERTICAL, HORIZONTAL, AND NMO VELOCITIES DERIVED FROM THE MAPPED MOVEOUT APPROXIMATIONS

Using the slowness projections 21 in the hyperbolic approximation 10, we obtain

$$\frac{1}{V_z} = u_{0x}^2 + u_{0z}^2, \quad \frac{1}{V_s} = \frac{1}{V_{nmo}^2} = u_{0x}^2 + u_{0z}^2.$$  \hfill (A-1)$$

For the shifted hyperbola approximation 13, we have

$$\frac{1}{V_z} = \frac{1}{S}[(S-1)u_{0x} + \sqrt{u_{0x}^2 + Su_{0z}^2}], \quad \frac{1}{V_s} = \frac{1}{S}[(S-1)u_{0x} + \sqrt{u_{0z}^2 + Su_{0z}^2}], \quad \frac{1}{V_{nmo}^2} = u_{0x}^2 + u_{0z}^2 + S - 1 \quad S^2$$

$$\times \left[-2u_{0x}u_{0z} - 2u_{0x}^2u_{0z}^2 + 4u_{0x}u_{0z}^2u_{0z}^2 + S(u_{0x}u_{0z}^2 + u_{0z}u_{0x}^2)\right]. \hfill (A-2)$$

From the rational approximation 15, we have

$$\frac{1}{V_z} = \frac{1}{u_{0x}^2 + u_{0z}^2 - 2\eta u_{0z}}, \quad \frac{1}{V_s} = \frac{1}{u_{0x}^2 + (1 + 2\eta)u_{0z}^2}, \quad \frac{1}{V_{nmo}^2} = \frac{1}{u_{0x}^2 + (1 + 2\eta)u_{0z}^2},$$

$$\frac{1}{V_z} = \frac{1}{u_{0x}^2 + u_{0z}^2 - 2\eta u_{0z}}, \quad \frac{1}{V_s} = \frac{1}{u_{0x}^2 + (1 + 2\eta)u_{0z}^2}, $$

$$\frac{1}{V_{nmo}^2} = \frac{1}{u_{0x}^2 + u_{0z}^2 - 2\eta u_{0z}}, \quad \frac{1}{V_{nmo}^2} = \frac{1}{u_{0x}^2 + (1 + 2\eta)u_{0z}^2}.$$  \hfill (A-3)

For the generalized moveout approximation 20, we obtain

$$\frac{1}{V_z} = \frac{1}{u_{0x}^2 + u_{0z}^2 - \frac{4u_{0x}u_{0z}}{u_{0x}^2 + Bu_{0z}^2 + S_1}}, \quad \frac{1}{V_s} = \frac{1}{u_{0x}^2 + u_{0z}^2 - \frac{4u_{0x}u_{0z}}{u_{0x}^2 + Bu_{0z}^2 + S_2}}, \quad \frac{1}{V_{nmo}^2} = \frac{1}{u_{0x}^2 + u_{0z}^2 - \frac{4u_{0x}u_{0z}}{u_{0x}^2 + Bu_{0z}^2 + S_2}}$$

$$\times \left[\frac{(2u_{0x}u_{0z} - u_{0x}^2 - u_{0z}^2) + Bu_{0x}(3u_{0x}u_{0z} - u_{0x}^2 - u_{0z}^2) + Cu_{0x}u_{0z}^2 + (2u_{0x}u_{0z} - u_{0x}^2 - u_{0z}^2)Bu_{0z}^2}{S_1(u_{0x}^2 + Bu_{0z}^2 + S_2)}\right]. \hfill (A-4)$$

with

$$S_1 = \sqrt{u_{0x}^4 + 2Bu_{0x}^2u_{0z}^2 + Cu_{0z}^4}, \quad S_2 = \sqrt{u_{0x}^4 + 2Bu_{0x}^2u_{0z}^2 + Cu_{0z}^4},$$

The parameters B and C are calculated from a horizontal ray in an homogeneous VTI medium and are given in equation 19.

REFERENCES


