Prestack phase-shift migration of separate offsets in laterally inhomogeneous media

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ABSTRACT

Using the stationary-phase method, prestack phase-shift migration is implemented one offset at a time. This separate-offset implementation allows for a Fourier (reasonably fast) wave-equation-type migration on data with irregular offset sampling. However, the separate-offset phase-shift migration, like its zero-offset counterpart, handles only vertically inhomogeneous media. Using the combination of the split-step and phase-shift-plus-interpolation (PSPI) approaches, the separate-offset phase-shift migration is extended to handle laterally inhomogeneous media. The cost of the separate-offset implementation is practically equivalent to that of the conventional zero-offset version. However, due to the lack of exact source and receiver ray-trajectory information in the separate-offset implementation, the combined split-step and PSPI handles only smooth lateral inhomogeneity. Specifically, it produces images equivalent to those resulting from smoothing the velocity model laterally over a window equal to the half offset. Thus, for zero-offset or laterally homogeneous media, the separate-offset migration is equivalent to any wave-equation-based migration. Errors might occur for finite-offset data in laterally inhomogeneous media. Such errors depend primarily on the strength of lateral inhomogeneity. Using this separate-offset phase-shift migration, accurate images of synthetic data of a model with large reflector dips and good images from real data from offshore Trinidad are obtained.

INTRODUCTION

Using the double-square-root (DSR) equation of Yilmaz (1979), the phase-shift approach of Gazdag (1978) can be implemented to image prestack data. However, its use in practice is rare; using the DSR equation in its original form requires that all offsets and common-midpoint (CMP) data be downward-continued simultaneously. The output of such a prestack migration process is a stacked image that does not contain moveout information that can be used for velocity analysis, or otherwise some notable artifacts will occur during integration of the offset-wavenumber axis because of the sparseness in its sampling. Popovici (1993) shows that such artifacts can be avoided by subsampling the offset-wavenumber axis so that the integration range does not include evanescent waves. However, now one can obtain moveout information from the DSR method by extracting angle gathers (Sava and Fomel, 2003).

Extending the prestack phase-shift migration to handle lateral inhomogeneity is accomplished using the phase-shift-plus-interpolation (PSPI) method introduced by Gazdag and Sguazzero (1984). Numerous other methods to extend the wave-equation (phase-shift) migration to laterally inhomogeneous media exist as well. Among these methods are the split-step approach of Stoffa et al. (1990) and the Fourier finite-difference method of Ristow and Ruhl (1994). Huang and Wu (1996) develop a family of methods for prestack phase-shift migration in laterally inhomogeneous media relying on the generalized screen propagator, in which the simplest form of it is the split-step approach.

In most prestack phase-shift migration methods developed, especially those dependent on the DSR equation, the offset axis must be converted to the wavenumber domain. This process often requires a regular and fine sampling of the offset axis, which can considerably increase the cost of the migration and limit its practical use on 3D data. Occasionally, azimuth-moveout operators (Biondi et al., 1998) and interpolation are used to regularize the offset axis. However, even these processes have their limitations and cost. There is no question that wave-equation migrations regularly produce better images than those obtained from a conventional Kirchhoff implementation, especially in complex media. However, the above-mentioned limitations associated with the phase-shift approach restrict its practical use in imaging large data sets.

Popovici (1993) devises an approach, based on DSR, to downward-continue separate offsets by evaluating the offset-wavenumber integral with the stationary-phase method. Determining the sta-
tionary point, however, required solving a sixth-order polynomial, which Popovici handles analytically. His numerical stationary-point solutions, according to his assessment, were relatively expensive, because they required evaluating too many integrals. In Alkhalifah (2000), I solve these problems by developing approximate, yet very accurate, analytical solutions for the stationary points in homogeneous media. Popovici also develops efficient numerical solutions for such stationary points in laterally homogeneous media. The result is an efficient separate-offset migration that costs about 10% more than its zero-offset counterpart.

In this paper, I extend the separate-offset phase-shift migration of Alkhalifah (2000) to handle smooth laterally inhomogeneous media. Initially, I use the split-step approach to correct for lateral inhomogeneity in the $x$-$\omega$ domain. The split-step method provides almost exact kinematic corrections for horizontal reflections, and approximate corrections for dipping events. Later, I combine the split-step approach with PSPI through a hybrid implementation. This hybrid approach results in reasonable correction for lateral inhomogeneity for both horizontal and dipping events. I apply the resulting separate-offset migration on synthetic data with large dips. Application of the method on the Trinidad data demonstrates its effectiveness.

**PRESTACK PHASE-SHIFT MIGRATION**

To perform the phase-shift migration in the prestack domain, the conventional zero-offset dispersion relation is replaced by a double-square-root (DSR) equation. Also, an additional Fourier transform over offset is needed.

For prestack migration, two imaging conditions apply: the zero-time ($t = 0$) and zero-offset ($h = 0$, where $h$ is the half offset) conditions. Thus, constant-velocity prestack migration (Yilmaz, 1979), with an image $g$ described in depth $z$, is given by

$$
g(t = 0, k_x h = 0,z) = \int d\omega \int dk_h e^{ik_z (\omega k_x h_0 + \omega) z} \times F(\omega, k_x, k_h, h = 0),$$

(1)

where $F(\omega, k_x, k_h, h = 0)$ is the 3D Fourier transform of the wavefield $f(t, x, h, z = 0)$ recorded at the surface

$$F(\omega, k_x, k_h, h = 0) = \int dt e^{-i\omega t} \int dx e^{ik_xt} \times \int dh e^{ik_h f(t, x, h, z = 0)},$$

$k_x$ is the midpoint wavenumber, and $k_h$ is the offset wavenumber. The depth wavenumber $k_z$ is given by the DSR equation, which, for isotropic 2D media, is given by

$$k_z = \frac{1}{2} \left[ \frac{\omega^2}{v^2} - (k_x - k_h)^2 \right]^{1/2} + \left[ \frac{\omega^2}{v^2} - (k_x + k_h)^2 \right]^{1/2}.$$  

(2)

Equation 1, with the proper values of $k_x$ and $k_h$, to avoid imaginary values of $k_z$, is used to do prestack migration. To obtain real values of $k_z$ that satisfy the downward-continuation ordinary-differential equation

$$\frac{d^2 W}{dz^2} = -k_z^2 W,$$  

(3)

$k$, and $k_h$ must satisfy

$$\frac{\omega}{v} \geq |k_x + k_h|$$

and

$$\frac{\omega}{v} \geq |k_x - k_h|.$$  

Both conditions are satisfied by insureing that

$$|k_x| + |k_h| \geq \frac{\omega}{v}.$$  

(4)

The DSR-based migration can be put in a form to allow for separate-offset migration of each constant-offset section (Popovici, 1993). Thus, for migration of a separate-offset section, $F(\omega, k_x, h_0, z = 0)$, equation 1 becomes

$$g(t = 0, k_x, h = 0, z) = \int d\omega F(\omega, k_x, h_0, z = 0) \times \int dk_h e^{ik_z (k_x h_0 + \omega) z},$$

(5)

where $h_0$ is the half offset of the section.

The cost of migrating separate offsets using equation 5 is close to the cost of migrating the whole data; the number of integrations in equations 1 and 5 are the same. However, the $k_x$ integral in equation 5 now can be evaluated using the stationary-phase method. As I show in Alkhalifah (2000), the stationary point for isotropic homogeneous media is obtained by finding the maximum (with respect to $k_x$) of the following equation:

$$T_h(k_h) = \frac{1}{2} \left[ \frac{\omega^2}{v^2} - (k_x - k_h)^2 + \frac{\omega^2}{v^2} - (k_x + k_h)^2 \right]$$

$$+ k_h h_0,$$

(6)

which corresponds to the terms in square brackets in equation 5. Replacing $k_x$ with $p_x$, and $k_h$ with $p_h$, in equation 6, where $p_x$ and $p_h$ are the horizontal and offset ray parameters, respectively, results in

$$T(p_h) = \frac{1}{2} \left[ \frac{1}{v^2} - (p_x - p_h)^2 + \frac{1}{v^2} - (p_x + p_h)^2 \right]$$

$$+ p_h h,$$

(7)

which is the frequency-independent version of equation 2. The maximum or minimum of $T$ and $T_h$ occurs at the same point, $k_x = op_x$, the stationary-phase point. The anisotropic version of $T$ is given by Alkhalifah (2000) and can be used in a similar way as the isotropic case.

**SEPARATE-OFFSET MIGRATION**

To implement prestack phase-shift migration, we need to evaluate the phase (or $T$) for which the data will be phase-shifted. Unlike zero-offset migration, where the offset ray parameter $p_h$ equals zero (and therefore, the phase shift can be determined directly), we must first obtain $p_h$ in separate-offset prestack migration and then use it to
evaluate the phase shift. The value of $p_{o0}$ can be determined numerically using any number of existing numerical methods (some based on evaluating derivatives), or through using analytical approximations (Alkhalifah, 2000). The efficiency in the numerical implementations is achieved by setting up numerous precomputed travelt ime tables at different stages of the migration. These tables are necessary to avoid redundancy in the calculations.

### A. Numerical solutions of $p_{o0}$

To implement prestack phase-shift migration efficiently in $v(z)$ media, we need to construct a table of $\tau(p,z)$ using $v(z)$ ray tracing, where $\tau$ is the vertical phase-shift factor for that particular ray, $p = \frac{k}{\lambda}$ is the ray parameter for that ray whether it is for the source or receiver, and $k$ is the corresponding horizontal wavenumber. Considering an initial lateral homogeneity assumption, this table is constructed once and is applicable everywhere. The stationary point is evaluated by finding

$$\max[T],$$

where

$$T = \frac{1}{2} [\tau(p_s + p_h,z) + \tau(p_s - p_h,z)] + 2p_{o0}h,$$

and

$$\tau(p,z) = \int_0^z p_s(p_\xi) d\xi.$$ 

(8)

By using an integral in equation (10), I am allowing velocity to vary with depth (or vertical time in this case).

### B. Phase calculations

Once the offset ray parameter $p_{o0}$ (or an approximation of it) is determined, it is then used, along with equation (9), to calculate traveltime (phase shifts). What makes zero-offset migration so trivial is that $p_{o0}$ is known in advance ($p_{o0} = 0$), and therefore, the travelt ime calculation can be done explicitly. For prestack migration, we must first estimate $p_{o0}$, then proceed with travelt ime calculations. The rest of the prestack phase-shift migration algorithm is exactly the same as its zero-offset counterpart.

Another table $\tilde{T}(p_{o0},z)$, consisting of $T$ for the solutions of equation 8 for given $p_{o0}, z$, and, offset, is constructed. The cost of a prestack migration of any offset is about 10% higher than that for a zero-offset algorithm, where $p_{o0} = 0$ is known, and thus we do not need to evaluate it. This relatively small additional cost reduces even further when large data sets are migrated and the cost of precomputing the traveltime tables become insignificant. These traveltime tables of stationary-phase solutions are constructed using rays traced from the surface all the way down to the imaged depth for vertically inhomogeneous media. Thus, we lack the exact phase-shift information necessary for a single depth step in laterally inhomogeneous media. Including a depth-step-based correction to the process will prove to be challenging.

### HANDLING LATERAL INHOMOGENEITY

Numerous methods exist to extend the conventional phase-shift migration to one that handles lateral velocity variation. Here, I investigate the possibility of using two of these methods: the split-step approach and the PSPI approach. I also look into combining the two methods in a hybrid approach.

#### The split-step approach

The split-step method for applying phase-shift migration on data from laterally inhomogeneous media is first introduced by Stoffa et al. (1990). It is one of the simplest ways to extend phase-shift migration to handle laterally inhomogeneous media. Popovici (1996) demonstrates the effectiveness of the method in extending the conventional prestack phase-shift migration to handle lateral inhomogeneity. In summary, the split-step approach provides the necessary correction required for horizontal reflections by phase-shifting the data in the $x-\omega$ domain by an amount necessary to place horizontal events accurately in depth.

Application of the split-step approach on the separate-offset phase-shift migration described by Alkhalifah (2000) is not straightforward. In the separate-offset approach, phase shifts up to a given depth (or vertical time) for a given velocity model are tabulated, so we need to convert these tabulated phase shifts to ones valid for single depth steps (see Figure 1). This is conveniently done by subtracting the travelt ime phase-shifts for a certain depth from that corresponding to the next depth step as follows:

$$\tilde{T}(p_{o0},\Delta z) = \tilde{T}(p_{o0},z) - \tilde{T}(p_{o0},z + \Delta z).$$

(11)

Now $\tilde{T}(p_{o0},\Delta z)$ represent the travelt ime phase shift required for downward continuation for a single depth step in vertically inhomogeneous media. This modification is needed because we have to convert the image from the wavenumber domain to the space ($x$) domain at each depth step. The conversion is necessary to apply the correction needed to accurately image the data in laterally inhomogeneous media. However, the new single-depth-step correction does not reflect the corrections applied to previous depths at their true downward-continued source and receiver locations. As shown in Figure 1, the single-depth-step correction is calculated from the difference in the total traveltimes to the surface for laterally homogeneous media. Because I am downward continuing single offsets, that data (unlike in Popovici, 1993) does not contain explicit source and receiver rayparameter information necessary to do proper source and receiver
corrections. Thus, the lateral inhomogeneity within this range between the source and receiver will not be accurately represented. Nevertheless, the accuracy depends on the degree of lateral inhomogeneity. After modifying the traveltime table, I use the algorithm shown in Figure 2a to implement the split-step method. The velocity \( v_w(z) \) is the velocity used for the downward continuation and it can be set to equal the average velocity at every depth \( z \).

The PSPI approach

Gazdag and Sguazzero (1984) develop an approach to extend the conventional phase-shift migration to handle lateral inhomogeneity. The method is based on downward continuing the data more than once using different laterally homogeneous velocity models, then interpolating the final image (in the \( x-\omega \) domain) from these downward-continued images. The number of downward continuations needed depends on the velocity model, and the cost of migration is usually high for complex velocity models.

Gazdag and Sguazzero (1984) initially use two laterally homogeneous velocity models as reference velocities for their interpolation method. In addition, they suggest that if the difference between the two velocities is large, additional velocities could be needed. To use PSPI in separate-offset migration, I develop traveltime tables for different laterally homogeneous models. As in the case for the split-step approach, I convert these traveltime tables to ones corresponding to single depth steps. Because linear interpolation is used between downward-continued images, artifacts will occur in the migration whenever the velocity used for each downward continuation causes cycle shifts between the downward-continued images larger than a half wavelength. Thus, for PSPI I have to reduce the vertical shift between the images that results from the downward continuation using different laterally homogeneous velocity models. This is accomplished by subtracting the traveltimes of the vertical plane.

\[
\Delta T(p, z) = \tilde{T}(p, z) - \tilde{T}(0, z). \tag{12}
\]

This step will effectively remove the influence of velocity on the horizontal component of the imaging operator. The subtracted horizontal-event phase shift is added once again after interpolation.

The hybrid approach

The split-step and PSPI approaches can help extend the phase-shift migration to handle laterally inhomogeneous media. However, as seen above, the two methods differ in their approaches, and thus differ in the result. Although the split-step method provides optimal results for horizontal events, the PSPI does a better job on the dipping events. A combination of the two methods, taking advantage of the features of each method, results in a better treatment for all dips. Thus, the basic idea (as demonstrated in Figure 2b) is to apply full split steps using different vertical varying velocities (for \( v_w(z) \)) and then interpolate between the images at each depth step.

In this hybrid approach, I allow for the downward continuation of up to three versions of the wavefield for three different vertically varying velocity models (instead of just the two in Figure 2b, shown for illustration purposes). One velocity model includes the minimum velocity at each depth step, the other includes the maximum velocity in each depth step, and the third includes the average velocity at every depth step. Thus, using the actual velocity, a split-step phase-shift correction is applied in the frequency-space domain to locate the horizontal reflectors accurately in depth. This is done for each of the three downward-continued wavefields. Finally, the image is linearly interpolated using the actual velocity model to obtain the final image at that depth (see Figure 2b).

Cost of the hybrid approach is governed by the cost of the PSPI step, which in turn depends on the number of downward-continued wavefields. Here, I suggest a maximum of three downward continuations for three different vertically varying velocity models. This will keep the cost down, yet provide a reasonable correction for most laterally variant velocity models. To speed up the migration, it is also possible to downward-continue one or two wavefields, instead of three. The downward continuation of one wavefield reduces this method to the split-step approach.

The smooth lateral-inhomogeneity assumption

For phase-shift migration to work for separate offsets, we must find the stationary-phase solutions of the tabulated phases based on the \( v(z) \) assumption. These tabulated phases correspond to the phase correction from the surface to the image depth in one step. As a result, using these tabulated phases, we lose the sense of lateral location within the source-receiver offset range. Thus, to accommodate lateral inhomogeneity in the split-step approach, I prefer to use a localized velocity computed by averaging the slowness of the medium for a certain depth step over the source-receiver offset range. However, one can implement lateral velocity smoothing for all offsets over the maximum source-receiver offset treated. In essence, I am smoothing laterally the original velocity over a window equal to the half offset for each source and receiver location. Errors resulting from such an approximation are about the same as the error that can occur using the Kirchhoff method and smoothing the velocity laterally over a window equal to the size of half the source-receiver offset. Thus, the method is valid mostly for smooth laterally varying media. However, the quality of the resulting images is similar to those of the downward-continuation-based migration.

The approach above is applicable to 3D data by substituting the midpoint and offset wavenumbers with wavenumber vectors containing elements representing the in-line and crossline (\( x \) and \( y \)) components. This is possible because the TI medium considered here has a vertical symmetry axis and thus is isotropic in the horizontal plane. Also, because the traveltimes are based on a local
Synthetic data examples

In the following synthetic and real data examples, I test the quality and accuracy of images produced by the separate-offset phase-shift approach.

Impulse response

The impulse response generated by inputting an impulse into a migration code provides valuable information on the sharpness and robustness of the migration. Figure 3 shows a velocity model with three prominent layers. All three layers include lateral velocity variation. The first layer includes also vertical velocity variation. The high-velocity middle layer should pose an interesting challenge to any migration code.

Figure 4 shows the impulse response of the separate-offset phase-shift migration due to an input pulse at 2.3 s. A pulse at this time (2.3 s) corresponds to a horizontal reflection in the third layer at a depth of about 1500 m. The input pulse has a peak frequency of 25 Hz. The input section containing the pulse was given an offset of 1500 m. The impulse response clearly displays the discontinuities resulting from the sharp velocity contrasts especially between the first and second layer. The impulse response also clearly maintains the high-frequency content of the original pulse. It is generally clean — a feature that will help provide good images in migration.

Synthetic model

In this example, I consider a simple model consisting of a horizontal and steeply dipping reflectors (Figure 5) embedded in a linear-velocity medium background (Alkhalifah, 1995). Velocity increases linearly as a function of the following equation:

\[ v(x,z) = 1500 + 0.3x + 1.1z, \]

where \( x \) corresponds to the horizontal distance from the origin in which velocity equals 1500 m/s, and \( z \) is depth measured from the surface. Figure 6 shows the synthetic data generated using Kirchhoff modeling from such a model for an offset of 400 m and 600 m. The offset-to-depth ratios for most of the reflector model exceed 1. The lateral velocity variation has clearly tilted the shape of the reflectors, giving us a dipping reflector appearance for the horizontal reflector.

Figure 7 shows the result of imaging the two sections using the separate-offset phase-shift migration. The reflectors in the two sections are overall accurately positioned both laterally and in depth. In conventional prestack phase-shift migrations, imaging the two sections alone is impossible; the full complement of offsets at a regular offset sampling grid is needed before such a migration is possible. Note that the cost of this separate-offset migration (including the full complement of offsets) is similar to the cost of any conventional prestack phase-migration method considering the same lateral velocity treatment. The main contribution of this migration is its ability in migrating individual offsets. This feature is especially important in imaging 3D data where coarse sampling of offset can considerably reduce the cost of migration. In fact, this feature is especially
important when applying prestack migration velocity analysis
where migration efficiency gained by relying on a few offsets is of
utmost importance.

The Trinidad example
A line of seismic data from Trinidad was acquired for Amoco
(now BP). The line spanned a distance of more than 50 km and I
show results from the application of separate-offset migration on
only about 9 km of the line. The offset extends beyond 6 km, which
provides ample information for anisotropy analysis. I use the aniso-

tropic results from the velocity-analysis technique described by
Alkhalifah (2005), and thus use the interval velocity and anisotropic
parameter \( \eta \) sections (Alkhalifah and Tsvankin, 1995) shown in Fig-

ure 8. The TAU imaging approach is basically a depth migration
mapped to the vertical time domain (Alkhalifah, 2003).

Using the anisotropy parameters in Figure 8, I apply an anisotrop-

ic prestack separate-offset migration. Figure 9 shows the stacked
output section after migrations up to a depth of 4.2 km. The image
here, despite the inherent smooth lateral approximation, compares
well with the result obtained by Alkhalifah (2005) using a Kirchhoff
method and shown in Figure 10. In fact, the separate-offset image
has better fault-reflection definition and continuity.

Figure 11 shows common-image gathers of the Trinidad line after
prestack separate-offset migration using only the interval velocity in
Figure 8 displayed every 40 CMPs. Residual moveout, especially at
the far offsets, are apparent with the familiar hockey-sticks effect ap-

pearing in many locations. Meanwhile, Figure 12 shows common-
image gathers from the same area after prestack separate-offset mi-
gation using the velocity and anisotropy parameter in Figure 8 dis-
played every 40 CMPs as well. The image gathers show a general
alignment of reflections with offset (more so than the isotropic case),
demonstrating the accuracy of the anisotropic velocity model and

Figure 8. (a) An interval velocity obtained from prestack TAU mi-
gration velocity analysis. (b) An interval \( \eta \) estimated using nonhy-
perbolic moveout analysis.

Figure 9. A stacked section of the Trinidad line after prestack sepa-
rate-offset migration using the interval velocity and \( \eta \) in Figure 8.

Figure 10. A stacked section of the Trinidad line after prestack Kirchhoff migration using the interval velocity and \( \eta \) in Figure 8.

Figure 11. Common-image gathers of the Trinidad line after prestack separate-offset migration using only the interval velocity in Figure 8 and ignoring the anisotropy. Image gathers are plotted every 40th
CMP.

Figure 12. Common-image gathers of the Trinidad line after prestack separate-offset migration using the interval velocity and \( \eta \) in Figure 8. Image gathers are again plotted every 40th CMP.
migration. Thus, anisotropy has a major influence on the migration for this data set, and thus should be included.

**CONCLUSIONS**

Prestack phase-shift migration for separate offsets allows for more flexible treatment of seismic data with irregular offset geometry. It also allows for faster phase-shift migrations of limited offsets. I extend the separate-offset migration to handle smooth lateral inhomogeneity. This migration, like other downward-continuation-based migrations, provides generally better images than those derived from the conventional Kirchhoff method, and yet is flexible enough to handle data with limited and irregular offset distribution. This feature is especially important for tasks like migration velocity analysis where efficient migration of portions of the data is needed. Good images were obtained for synthetic data with strong lateral inhomogeneity and large dips. Application on real data from Trinidad demonstrated the capability of this method in producing accurate images of the subsurface.

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